Published by Institute of Physics Publishing for SISSA

Received: *September 11, 2006*
Revised: *February 8, 2007*
Accepted: *February 26, 2007*
Published: *March 8, 2007*

The one-loop renormalization of the gauge sector in the θ-expanded noncommutative standard model

Maja Burić and Voja Radovanović

Faculty of Physics, University of Belgrade,
P. O. Box 368, 11001 Belgrade, Serbia
E-mail: majab@phy.bg.ac.yu, rvoja@phy.bg.ac.yu

Josip Trampetić

Rudjer Bošković Institute, Theoretical Physics Division,
P.O.Box 180, 10002 Zagreb, Croatia, and
Max Planck Institut für Physik,
Föhringer Ring 6, D-80805 München, Germany
E-mail: josipt@rex.irb.hr

Abstract: In this paper we construct a version of the standard model gauge sector on noncommutative space-time which is one-loop renormalizable to first order in the expansion in the noncommutativity parameter θ. The one-loop renormalizability is obtained by the Seiberg-Witten redefinition of the noncommutative gauge potential for the model containing the usual six representations of matter fields of the first generation.

Keywords: Non-Commutative Geometry, Standard Model, Renormalization Regularization and Renormalons.

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1. Introduction

The interest to formulate a consistent quantum field theory on noncommutative space comes, besides from string theory, also from mathematics [1] and from phenomenology. After the initial noncommutative standard model (NCSM) by Connes [2] many other models were proposed, differing in their physical properties such as particle content, additional symmetries, grand unification scheme, etc.

There are two main approaches to define gauge theories on the canonical noncommutative space. One possibility, extensively analyzed in the literature [3, 4], is to replace the ordinary product in the Lagrangian by the Moyal-Weyl $\star$-product; it is well defined owing to associativity and the trace property of the $\star$-product. Using this prescription, however, only $\mathrm{U}(N)$ gauge theories can be consistently defined and the group representations are restricted to the fundamental and the adjoint. This implies in particular the quantization of the electric charge which takes values in $\{\pm 1, 0\}$. In perturbative quantization, the interaction vertices obtain additional phase factors in comparison with commutative theory, and this leads to the well-know UV/IR mixing.

A slightly different and nonequivalent representation is the so-called θ-expanded approach. A consequence of the requirement that the gauge algebra closes on noncomutative fields is that the fields are enveloping algebra-valued. Using the Seiberg-Witten map, which is also an expansion in the noncommutativity parameter θ, noncommutative fields are expressed in terms of their commutative counterparts [5, 6]. There are therefore two symmetries in the theory: commutative gauge symmetry which is manifest in each order in θ, and noncommutative gauge symmetry which relates different orders and exists only after the summation. Commutative gauge theory is recovered in the limit $\theta \to 0$, which, in this case, is smooth. Assuming that noncommutativity is small and considering the theory as effective, the leading noncommutative effects can be calculated by truncation to linear order in θ. The major advantage of this approach is that models with any gauge group and any particle content can be constructed.

There is a number of versions of the noncommutative standard model in the θ- expanded approach [7 − 10] and in the other approaches too [11, 12]. The argument of renormalizability was previously not included in the construction because it was believed that field theories on noncommutative Minkowski space were not renormalizable in general [13, 14]. However, a recent result on the one-loop renormalizability of the θ-expanded noncommutative SU(N) gauge theory opens different perspectives [15 − 17]. In particular, a recent positive result [17] was our initial motivation to re-examine the noncommutative standard model gauge sector. Of course, renormalizability in linear order does not mean renormalizability of the complete theory, but one can expect that the additional Ward identities, which correspond to the full noncommutative symmetry and relate different orders, might help. In this paper we show that it is possible to construct a version of the NCSM gauge sector which is one-loop renormalizable to first order in θ. In addition, it has already been proved that the theory is anomaly free whenever its commutative counterpart is anomaly free [18].

An important point in our consideration is that the SW map is not unique. Therefore, the SW map in fact gives a class of 'truncated' gauge theories, no theory a priori preferred. We show that the requirement of renormalizability pinpoints one of them as 'physical'.

A reason to focus on the gauge sector of the NCSM is the possibility to detect, in the forthcoming experiments at LHC, decays which are forbidden in the SM [9, 19], like $Z \to \gamma\gamma$, and/or to find deviations with respect to the SM-predicted angular distributions of the differential cross section in $\bar{f}f \to \gamma\gamma$, etc. scattering [20, 21]. In all of these transitions the so-called triple gauge boson (TGB) couplings contribute. Clearly, from the perspective of the safe usage of noncommutativity-induced corrections to the TGB couplings in further phenomenological analysis of the above processes, it is important to prove the regular behavior of these interactions with respect to the one-loop renormalizability. Signatures of noncommutativity in experimental particle physics were discussed in the literature from the point of view of collider physics [22]. Decays which are strictly forbidden in the SM by angular momentum conservation and Bose statistics, known as the Landau-Pomeranchuk-Yang theorem, as well as noncommutativity from neutrino astrophysics and neutrino physics were discussed in [9, 19] and [23], respectively.

The plan of the paper is the following. In section 2 we briefly review the ingredients of the NCSM relevant to this work. In section 3 the renormalizability of the NCSM gauge sector is worked out; in Subsection 3.3 the counter terms and the final Lagrangian are explicitly given. Section 4 is devoted to the discussion of the results and to the concluding remarks.

JHEP03(2007)030

2. Noncommutative standard model

2.1 General considerations

The noncommutative space which we consider is the flat Minkowski space, generated by four hermitian coordinates $\widehat{x}^{\mu}$ which satisfy the commutation rule

$$[\widehat{x}^{\mu}, \widehat{x}^{\nu}] = i\theta^{\mu\nu} = \text{const.} \tag{2.1}$$

The algebra of the functions $\widehat{\phi}(\widehat{x})$, $\widehat{\chi}(\widehat{x})$ on this space can be represented by the algebra of the functions $\widehat{\phi}(x)$, $\widehat{\chi}(x)$ on the commutative $\mathbf{R}^4$ with the Moyal-Weyl multiplication:

$$\widehat{\phi}(x) \star \widehat{\chi}(x) = e^{\frac{i}{2}\,\theta^{\mu\nu}\frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial y^{\nu}}}\,\widehat{\phi}(x)\widehat{\chi}(y)|_{y\to x}\ . \tag{2.2}$$

It is possible to represent the action of an arbitrary Lie group G (with the generators denoted by T^a) on noncommutative space. In analogy to the ordinary case, one introduces the gauge parameter $\widehat{\Lambda}(x)$ and the vector potential $\widehat{V}_{\mu}(x)$. The main difference is that the noncommutative $\widehat{\Lambda}$ and $\widehat{V}_{\mu}$ cannot take values in the Lie algebra $\mathcal{G}$ of the group G: they are enveloping algebra-valued. The noncommutative gauge field strength $\widehat{F}_{\mu\nu}$ is defined in the usual way

$$\widehat{F}_{\mu\nu} = \partial_{\mu}\widehat{V}_{\nu} - \partial_{\nu}\widehat{V}_{\mu} - i(\widehat{V}_{\mu}\star\widehat{V}_{\nu} - \widehat{V}_{\nu}\star\widehat{V}_{\mu}). \tag{2.3}$$

There is, however, a relation between the noncommutative gauge symmetry and the commutative one: it is given by the Seiberg-Witten (SW) mapping [5]. Namely, the matter fields $\widehat{\phi}$, the gauge fields $\widehat{V}_{\mu}$, $\widehat{F}_{\mu\nu}$ and the gauge parameter $\widehat{\Lambda}$ can be expanded in the noncommutative $\theta^{\mu\nu}$ and in the commutative V_{μ} and $F_{\mu\nu}$. This expansion coincides with the expansion in the generators of the enveloping algebra of $\mathcal{G}$, $\{T^a,\ :T^aT^b:,\ :T^aT^bT^c:\}$; here $:\ :$ denotes the symmetrized product. The SW map is obtained as a solution to the gauge-closing condition of infinitesimal (noncommutative) transformations. The expansions of the NC vector potential and of the field strength, up to first order in θ, read

$$\widehat{V}_{\rho}(x) = V_{\rho}(x) - \frac{1}{4}\,\theta^{\mu\nu}\,\{V_{\mu}(x), \partial_{\nu}V_{\rho}(x) + F_{\nu\rho}(x)\} + \ldots. \tag{2.4}$$

$$\widehat{F}_{\rho\sigma} = F_{\rho\sigma} + \frac{1}{4}\theta^{\mu\nu}\Big(2\{F_{\mu\rho}, F_{\nu\sigma}\} - \{V_{\mu}, (\partial_{\nu} + D_{\nu})F_{\rho\sigma}\}\Big) + \ldots \tag{2.5}$$

D_{μ} is the commutative covariant derivative.

The solution for the SW map given above is not unique. As it was shown in [13, 24], along with (2.5) all expressions $\widehat{V}'_{\mu}$, $\widehat{F}'_{\mu\nu}$ of the form

$$\widehat{V}'_{\mu} = \widehat{V}_{\mu} + X_{\mu}, \quad \widehat{F}'_{\mu\nu} = \widehat{F}_{\mu\nu} + D_{\mu}X_{\nu} - D_{\nu}X_{\mu}, \tag{2.6}$$

are solutions to the closing condition to linear order, if X_{μ} is a gauge covariant expression linear in θ, otherwise arbitrary. One can think of this transformation as of a redefinition of the fields V_{μ} and $F_{\mu\nu}$.

Taking the action of the noncommutative gauge theory

$$S = -\frac{1}{2}\mathrm{Tr}\int d^4x\,\widehat{F}_{\mu\nu}\star\widehat{F}^{\mu\nu}, \tag{2.7}$$

and expanding the fields as in (2.4)– (2.5) and the $\star$-product in θ, we obtain the expression

$$S = -\frac{1}{2}\mathrm{Tr}\int d^4x\, F_{\mu\nu}F^{\mu\nu} + \theta^{\mu\nu}\,\mathrm{Tr}\int d^4x\,\Big(\frac{1}{4}F_{\mu\nu}F_{\rho\sigma} - F_{\mu\rho}F_{\nu\sigma}\Big)F^{\rho\sigma}, \tag{2.8}$$

which is the starting point for the analysis of θ-expanded noncommutative gauge models. The action consists of two terms. The first term is the ordinary commutative action, and the second gives additional interactions which describe noncommutativity in the leading order in θ. In order to take into account the non uniqueness of the expansions (2.4)–(2.5), one should also add terms which correspond to the freedom (2.6). In the action this amounts to

$$S' = S - \mathrm{Tr}\int d^4x F^{\mu\nu}D_\mu X_\nu. \tag{2.9}$$

The additional terms which could be included in the Lagrangian (2.8), that is those linear in θ and of correct dimension are,

$$F^{\mu\nu}D_\mu X_\nu = F^{\mu\nu}D_\mu\left(b_1\,\theta^{\rho\sigma}D_\nu F_{\rho\sigma} + b_2\,\theta^{\rho}{}_{\nu}D^\sigma F_{\rho\sigma} + b_3\,\theta^{\rho\sigma}D_\rho F_{\nu\sigma}\right). \tag{2.10}$$

Out of these three terms the second vanishes owing to its symmetry-antisymmetry properties. The third term can be transformed into the first one using the Bianchi identities [1].

In summary, the freedom due to the SW field redefinitions reduces to the possibility to add one term, ΔS, to the original Lagrangian:

$$\Delta S = -2b\,\theta^{\rho\sigma}\,\mathrm{Tr}\int d^4x\, F^{\mu\nu}D_\mu D_\nu F_{\rho\sigma} = b\,\theta^{\rho\sigma}\,\mathrm{Tr}\int d^4x\, F^{\mu\nu}F_{\mu\nu}F_{\rho\sigma}. \tag{2.11}$$

Writing $b = -\frac{1}{4} + \frac{a}{4}$, we obtain the following general form of the noncommutative gauge field action:

$$S = -\frac{1}{2}\mathrm{Tr}\int d^4x\, F_{\mu\nu}F^{\mu\nu} + \theta^{\mu\nu}\,\mathrm{Tr}\int d^4x\,(\frac{a}{4}\,F_{\mu\nu}F_{\rho\sigma} - F_{\mu\rho}F_{\nu\sigma})F^{\rho\sigma}. \tag{2.12}$$

The coefficient a is going to be fixed by the requirement of renormalizability in the next section.

2.2 $\mathrm{U(1)_Y \otimes SU(2)_L \otimes SU(3)_C}$

The discussion given above was a general one, without any specification of the gauge group G or of its representations. However, as the θ-linear term in the action includes the trace of the product of three group generators, it is obvious that the action is a representation-dependent quantity. In the commutative case, the action contains only the trace of the product of two generators which is up to normalization the same for all group representations, $\mathrm{Tr}\,T^aT^b \sim \delta^{ab}$ (if we assume the usual properties of G, i.e. that it is semisimple, compact, etc.). But in (2.12) we have a factor $\mathrm{Tr}\,\{T^a, T^b\}T^c \sim d^{abc}$. One could perhaps

[1] One could in principle also add the parity violating terms. There are two independent expressions: $F^{\mu\nu}D_\mu\epsilon^{\rho\sigma\alpha\beta}\theta_{\alpha\beta}D_\nu F_{\rho\sigma}$ and $F^{\mu\nu}D_\mu\epsilon^{\nu\sigma\alpha\beta}\theta_{\rho\beta}D^\rho F_{\alpha\sigma}$. These terms violate parity if one assumes that $\theta^{\mu\nu}$ is invariant under parity; compare, however, with [8]. We shall not discuss such a possibility in this article.

assume that, as the field strength transforms according to the adjoint representation, the symmetric coefficients d^{abc} are given in that representation. However, when the matter fields are included, other representations of G are present too, and therefore the expression (2.12) is ambiguous.

To start the discussion of the gauge field action-dependence on the gauge group and/or on its representation, we use the most general form of the action, [8]:

$$S_{cl} = -\frac{1}{2}\int d^4x \sum_{\mathcal{R}} C_{\mathcal{R}} \mathrm{Tr}\left(\mathcal{R}(\widehat{F}_{\mu\nu}) * \mathcal{R}(\widehat{F}^{\mu\nu})\right). \tag{2.13}$$

The sum is, in principle, taken over all irreducible representations $\mathcal{R}$ of G with arbitrary weights $C_{\mathcal{R}}$. Of course, for the gauge group G we take $\mathrm{U(1)_Y} \otimes \mathrm{SU(2)_L} \otimes \mathrm{SU(3)_C}$. To relate the action (2.12) to the usual action of the commutative standard model, we make the decompositions

$$V_\mu = g'\mathcal{A}_\mu \mathcal{R}(Y) + gB^i_\mu \mathcal{R}(T^i_L) + g_S G^a_\mu \mathcal{R}(T^a_S), \tag{2.14}$$
$$F_{\mu\nu} = g' f_{\mu\nu} \mathcal{R}(Y) + gB^i_{\mu\nu} \mathcal{R}(T^i_L) + g_S G^a_{\mu\nu} \mathcal{R}(T^a_S). \tag{2.15}$$

The $\mathcal{R}(Y)$, $\mathcal{R}(T^i_L)$, $\mathcal{R}(T^a_S)$ denote the representations of the group generators Y, T^i_L and T^a_S of $\mathrm{U(1)_Y}$, $\mathrm{SU(2)_L}$ and $\mathrm{SU(3)_C}$, respectively; the group indices run as $i, j = 1, \ldots 3$ and $a, b = 1, \ldots 8$. According to [8], we take that $C_{\mathcal{R}}$ are nonzero only for the particle representations which are present in the standard model. Then from (2.13) we obtain the expression for the θ-independent part of the Lagrangian

$$\begin{aligned}\mathcal{L}_{SM} = &-\frac{1}{2}g'^2 \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2) d(\mathcal{R}_3) \mathcal{R}_1(Y)\mathcal{R}_1(Y)\, f_{\mu\nu} f^{\mu\nu} \\ &-\frac{1}{2}g^2 \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_3) \mathrm{Tr}\,(\mathcal{R}(T^i_L)\mathcal{R}(T^j_L))\, B^i_{\mu\nu} B^{\mu\nu j} \\ &-\frac{1}{2}g_S^2 \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2) \mathrm{Tr}\,(\mathcal{R}(T^a_S)\mathcal{R}(T^b_S))\, G^a_{\mu\nu} G^{\mu\nu b},\end{aligned} \tag{2.16}$$

where $d(\mathcal{R})$ denotes the dimension of the representation $\mathcal{R}$. Identifying (2.16) with the SM Lagrangian, we find that the weights have to be constrained to match the coupling constants in the standard model in the following way [7 − 9]:

$$\frac{1}{2g'^2} = \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2) d(\mathcal{R}_3) \mathcal{R}_1(Y)^2, \tag{2.17}$$
$$\frac{1}{g^2}\frac{\delta^{ij}}{2} = \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_3) \mathrm{Tr}\,(\mathcal{R}(T^i_L)\mathcal{R}(T^j_L)), \tag{2.18}$$
$$\frac{1}{g_S^2}\frac{\delta^{ab}}{2} = \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2) \mathrm{Tr}\,(\mathcal{R}(T^a_S)\mathcal{R}(T^b_S)). \tag{2.19}$$

	$\mathrm{SU(3)_C}$	$\mathrm{SU(2)_L}$	$\mathrm{U(1)_Y}$	$\mathrm{U(1)_Q}$	$\mathrm{T_3}$
e_R	**1**	**1**	-1	-1	0
$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	**1**	**2**	$-1/2$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
u_R	**3**	**1**	$2/3$	$2/3$	0
d_R	**3**	**1**	$-1/3$	$-1/3$	0
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	**3**	**2**	$1/6$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	**1**	**2**	$1/2$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$

Table 1: Matter fields of the first generation. Electric charge is given by the Gell-Mann-Nishijima relation $Q = T_3 + Y$.

The noncommutative correction, that is the θ-linear part of the Lagrangian, reads

$$\begin{aligned}
\mathcal{L}^\theta &= \sum \mathcal{L}^\theta_i = g'^3 \kappa_1 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} f_{\rho\sigma} f^{\rho\sigma} - f_{\mu\rho} f_{\nu\sigma} f^{\rho\sigma} \right) \\
&+ g^3 \kappa_4^{ijk} \theta^{\mu\nu} \left(\frac{a}{4} B^i_{\mu\nu} B^j_{\rho\sigma} B^{\rho\sigma k} - B^i_{\mu\rho} B^j_{\nu\sigma} B^{\rho\sigma k} \right) \\
&+ g_S^3 \kappa_5^{abc} \theta^{\mu\nu} \left(\frac{a}{4} G^a_{\mu\nu} G^b_{\rho\sigma} G^{\rho\sigma c} - G^a_{\mu\rho} G^b_{\nu\sigma} G^{\rho\sigma c} \right) \\
&+ g' g^2 \kappa_2 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} B^i_{\rho\sigma} B^{\rho\sigma i} - f_{\mu\rho} B^i_{\nu\sigma} B^{\rho\sigma i} + c.p. \right) \\
&+ g' g_S^2 \kappa_3 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} G^a_{\rho\sigma} G^{\rho\sigma a} - f_{\mu\rho} G^a_{\nu\sigma} G^{\rho\sigma a} + c.p. \right),
\end{aligned} \tag{2.20}$$

where the *c.p.* in (2.20) denotes the addition of the terms obtained by a cyclic permutation of fields without changing the positions of indices. The couplings in (2.20) are defined as follows:

$$\kappa_1 = \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2) d(\mathcal{R}_3) \mathcal{R}_1(Y)^3, \tag{2.21}$$

$$\kappa_2 \delta^{ij} = \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_3) \mathcal{R}_1(Y) \mathrm{Tr}\,(\mathcal{R}_2(T^i_L) \mathcal{R}_2(T^j_L)), \tag{2.22}$$

$$\kappa_3 \delta^{ab} = \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2) \mathcal{R}_1(Y) \mathrm{Tr}\,(\mathcal{R}_3(T^a_S) \mathcal{R}_3(T^b_S)), \tag{2.23}$$

$$\kappa_4^{ijk} = \frac{1}{2} \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_3) \mathrm{Tr}\,(\{\mathcal{R}_2(T^i_L), \mathcal{R}_2(T^j_L)\} \mathcal{R}_2(T^k_L)), \tag{2.24}$$

$$\kappa_5^{abc} = \frac{1}{2} \sum_{\mathcal{R}} C_{\mathcal{R}} d(\mathcal{R}_2) \mathrm{Tr}\,(\{\mathcal{R}_3(T^a_S), \mathcal{R}_3(T^b_S)\} \mathcal{R}_3(T^c_S)). \tag{2.25}$$

Let us discuss the dependence of $\kappa_1, \ldots, \kappa_5$ on the representations of matter fields. For the first generation of the standard model there are six such representations, summarized in table 1; they produce six independent constants $C_{\mathcal{R}}$[2]. These constants are already constrained by the three relations (2.17)– (2.19). The couplings $\kappa_1, \ldots, \kappa_5$ given by (2.21)–(2.25) also depend on $C_{\mathcal{R}}$. However, one can immediately verify that $\kappa_4^{ijk} = 0$. This follows from the fact that the symmetric coefficients d^{ijk} of SU(2) vanish for all irreducible representations. We shall in addition take that $\kappa_5^{abc} = 0$. The argument for this assumption is related to the invariance of thecolour sector of the SM under charge conjugation. Although apparently in table 1 one has only the fundamental representation **3** of $\mathrm{SU}(3)_{\mathrm{C}}$, there are in fact both **3** and $\bar{\mathbf{3}}$ representations with the same weights, $C_{\mathbf{3}} = C_{\bar{\mathbf{3}}}$. In the Lagrangian this corresponds to writing each minimally-coupled quark terms a half of the sum of the original and the charge-conjugated terms. Since the symmetric coefficients for the **3** and $\bar{\mathbf{3}}$ representations satisfy $d_{\bar{\mathbf{3}}}^{abc} = -d_{\mathbf{3}}^{abc}$, we obtain

$$\kappa_5^{abc} = C_{\mathbf{3}} d_{\mathbf{3}}^{abc} + C_{\bar{\mathbf{3}}} d_{\bar{\mathbf{3}}}^{abc} = 0. \tag{2.26}$$

We are left only with three non vanishing couplings, κ_1, κ_2 and κ_3, depending on six constants $C_1, \ldots, C_6$ (indices $1, \ldots, 6$ enumerate the representations as they are given in table 1):

$$\begin{aligned}
\kappa_1 &= -C_1 - \frac{1}{4}\,C_2 + \frac{8}{9}\,C_3 - \frac{1}{9}\,C_4 + \frac{1}{36}\,C_5 + \frac{1}{4}\,C_6\,, \\
\kappa_2 &= -\frac{1}{4}\,C_2 + \frac{1}{4}\,C_5 + \frac{1}{4}\,C_6\,, \\
\kappa_3 &= +\frac{1}{3}\,C_3 - \frac{1}{6}\,C_4 + \frac{1}{6}\,C_5\,.
\end{aligned} \tag{2.27}$$

There are three relations among C_i's:

$$\begin{aligned}
\frac{1}{g'^2} &= 2\,C_1 + C_2 + \frac{8}{3}\,C_3 + \frac{2}{3}\,C_4 + \frac{1}{3} C_5 + C_6\,, \\
\frac{1}{g^2} &= C_2 + 3\,C_5 + C_6\,, \\
\frac{1}{g_s^2} &= C_3 + C_4 + 2\,C_5\,,
\end{aligned} \tag{2.28}$$

in effect representing three consistency conditions imposed on (2.12) in a way to match the SM action at zeroth order in θ. Note that detailed discussions about the solutions of the system of three equations (2.27) and six unequations $C_i > 0$, satisfying (2.28), are given in [9]. Our classical noncommutative action reads

$$S_{cl} = S_{SM} + S^{\theta}, \tag{2.29}$$

[2]We assume that $C_{\mathcal{R}} > 0$; therefore the six $C_{\mathcal{R}}$'s were denoted by $\frac{1}{g_i^2}$, $i = 1, \ldots, 6$, in [7, 9].

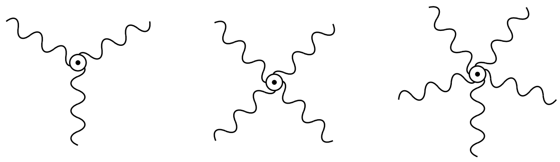

Figure 1: θ-vertices

with

$$S^\theta = \sum_{i=1}^{3} S_i^\theta = g'^3\kappa_1\theta^{\mu\nu}\int d^4x\left(\frac{a}{4}f_{\mu\nu}f_{\rho\sigma}f^{\rho\sigma} - f_{\mu\rho}f_{\nu\sigma}f^{\rho\sigma}\right)$$
$$+g'g^2\kappa_2\theta^{\mu\nu}\int d^4x\left(\frac{a}{4}f_{\mu\nu}B^i_{\rho\sigma}B^{\rho\sigma i} - f_{\mu\rho}B^i_{\nu\sigma}B^{\rho\sigma i} + c.p.\right)$$
$$+g'g_S^2\kappa_3\theta^{\mu\nu}\int d^4x\left(\frac{a}{4}f_{\mu\nu}G^a_{\rho\sigma}G^{\rho\sigma a} - f_{\mu\rho}G^a_{\nu\sigma}G^{\rho\sigma a} + c.p.\right). \qquad (2.30)$$

The noncommutative couplings introduce additional vertices, as depicted in figure 1. For simplicity, we do not distinguish the gauge fields $\mathcal{A}_\mu$, B^i_μ and G^a_μ by different types of lines: the dependence on the fields is not difficult to trace.

The term S_1^θ in (2.30) is one-loop renormalizable to linear order in θ [17] since the one-loop correction to the S_1^θ is of the second order in θ. We need to investigate only the renormalizability of remaining S_2^θ and S_3^θ parts of the action (2.30).

3. One-loop renormalizability

3.1 Effective action

We compute the divergences in the one-loop effective action using the background-field method [25, 26]. As we have already explained many details of similar calculations [14], here we just introduce the notation. Let the classical action be given by $S_{cl}[\phi]$; in our case, the fields are, $\phi_A = (\mathcal{A}_\mu,\, B^i_\mu,\, G^a_\mu)$. To quantize, one performs the functional integral. The integral over the quantum fields, $\mathbf{\Phi}_A$, can be calculated in the saddle-point approximation around the classical (background) configuration, denoted also by ϕ_A. The effective action is

$$\Gamma[\phi] = S_{cl}[\phi] + \Gamma^{(1)}[\phi]. \qquad (3.1)$$

The first quantum correction to the one-loop effective action $\Gamma^{(1)}[\phi]$, is given by

$$\Gamma^{(1)}[\phi] = \frac{i}{2}\log\det S^{(2)}_{cl}[\phi] = \frac{i}{2}\mathrm{Tr}\,\log S^{(2)}_{cl}[\phi]. \qquad (3.2)$$

In (3.2) the $S^{(2)}_{cl}[\phi]$ is the second functional derivative of the classical action,

$$S^{(2)}_{cl}[\phi] = \frac{\delta^2 S_{cl}}{\delta\phi_A\delta\phi_B}. \qquad (3.3)$$

In the case of the polynomial interactions as we have in (2.30), one can find $S_{cl}^{(2)}$ simply by splitting the fields into the classical-background plus the quantum-fluctuation parts, that is, $\phi_A \to \phi_A + \mathbf{\Phi}_A$, and by computing the terms quadratic in the quantum fields. For the action (2.12), the classical Lagrangian reads

$$\begin{aligned}
\mathcal{L}_{cl} &= \mathcal{L}_{SM} + \sum \mathcal{L}_i^\theta \\
&= -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} B^i_{\mu\nu} B^{\mu\nu i} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} \\
&\quad + g'^3 \kappa_1 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} f_{\rho\sigma} f^{\rho\sigma} - f_{\mu\rho} f_{\nu\sigma} f^{\rho\sigma} \right) \\
&\quad + g' g^2 \kappa_2 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} B^i_{\rho\sigma} B^{\rho\sigma i} - f_{\mu\rho} B^i_{\nu\sigma} B^{\rho\sigma i} + c.p. \right) \\
&\quad + g' g_S^2 \kappa_3 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} G^a_{\rho\sigma} G^{\rho\sigma a} - f_{\mu\rho} G^a_{\nu\sigma} G^{\rho\sigma a} + c.p. \right) .
\end{aligned} \tag{3.4}$$

Writing the *c.p.* terms in (3.4) explicitly, we obtain

$$\begin{aligned}
\mathcal{L}_{cl} = & -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} B^i_{\mu\nu} B^{\mu\nu i} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} \qquad (3.5) \\
& + g'^3 \kappa_1 \theta^{\mu\nu} \left(\frac{a}{4} f_{\mu\nu} f_{\rho\sigma} f^{\rho\sigma} - f_{\mu\rho} f_{\nu\sigma} f^{\rho\sigma} \right) \\
& + g' g^2 \kappa_2 \theta^{\mu\nu} (\frac{a}{4} f_{\mu\nu} B^i_{\rho\sigma} B^{\rho\sigma i} - 2 f_{\mu\rho} B^i_{\nu\sigma} B^{\rho\sigma i} + \frac{a}{2} f_{\rho\sigma} B^i_{\mu\nu} B^{\rho\sigma i} - f_{\rho\sigma} B^i_{\mu\rho} B^{\nu\sigma i}) \\
& + g' g_S^2 \kappa_3 \theta^{\mu\nu} (\frac{a}{4} f_{\mu\nu} G^a_{\rho\sigma} G^{\rho\sigma a} - 2 f_{\mu\rho} G^a_{\nu\sigma} G^{\rho\sigma a} + \frac{a}{2} f_{\rho\sigma} G^a_{\mu\nu} G^{\rho\sigma a} - f_{\rho\sigma} G^a_{\mu\rho} G^{\nu\sigma a}) ,
\end{aligned}$$

the classical Lagrangian which we are using next in the renormalization procedure.

3.2 Interaction vertices

In order to fix the quantum gauge symmetry, we have to add the gauge-fixing term to the Lagrangian (3.5). The gauge-fixing term is added to the θ-independent part in the usual way, [26, 14]. After making the splitting

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \mathbf{A}_\mu, \quad B^i_\mu \to B^i_\mu + \mathbf{B}^i_\mu, \quad G^a_\mu \to G^a_\mu + \mathbf{G}^a_\mu, \tag{3.6}$$

we obtain for the quadratic part of the action (3.5):

$$\frac{1}{2} \left(\mathbf{A}_\alpha \, \mathbf{B}^i_\alpha \, \mathbf{G}^a_\alpha \right) \begin{pmatrix} g^{\alpha\beta} \Box + M^{\alpha\beta} & * & * \\ * & g^{\alpha\beta} \delta^{ij} \Box + V^{\alpha\beta;ij} & 0 \\ * & 0 & g^{\alpha\beta} \delta^{ab} \Box + W^{\alpha\beta ab} \end{pmatrix} \begin{pmatrix} \mathbf{A}_\beta \\ \mathbf{B}^j_\beta \\ \mathbf{G}^b_\beta \end{pmatrix} . \tag{3.7}$$

In (3.7), $*$ stands for the terms which will not contribute to linear order: they give higher-order corrections. The first matrix element in (3.7) is given by $M^{\alpha\beta} = \overleftarrow{\partial_\mu} M^{\mu\alpha,\nu\beta}(x) \overrightarrow{\partial_\nu}$, where

$$\begin{aligned}
M^{\mu\rho,\nu\sigma} &= \frac{1}{2} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \theta^{\alpha\beta} f_{\alpha\beta} \\
&\quad + g^{\mu\nu} (\theta^{\alpha\rho} f^\sigma{}_\alpha + \theta^{\alpha\sigma} f^\rho{}_\alpha) + g^{\rho\sigma} (\theta^{\alpha\mu} f^\nu{}_\alpha + \theta^{\alpha\nu} f^\mu{}_\alpha) \\
&\quad - g^{\mu\sigma} (\theta^{\alpha\rho} f^\nu{}_\alpha + \theta^{\alpha\nu} f^\rho{}_\alpha) - g^{\nu\rho} (\theta^{\alpha\sigma} f^\mu{}_\alpha + \theta^{\alpha\mu} f^\sigma{}_\alpha) \\
&\quad + \theta^{\mu\rho} f^{\nu\sigma} + \theta^{\nu\sigma} f^{\mu\rho} - \theta^{\rho\sigma} f^{\mu\nu} - \theta^{\mu\nu} f^{\rho\sigma} - \theta^{\nu\rho} f^{\mu\sigma} - \theta^{\mu\sigma} f^{\nu\rho} .
\end{aligned} \tag{3.8}$$

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The structure of $V^{\alpha\beta;ij}$ is as follows:

$$V^{\alpha\beta;ij} = (N_1 + N_2 + T_1 + T_2 + T_3)^{\alpha\beta;ij}. \tag{3.9}$$

The operators N_1 and N_2 come from the commutative 3-vertex and 4-vertex interactions:

$$\begin{aligned}
(N_1)^{ij}_{\alpha\beta} &= -2ig_{\alpha\beta}(B_\mu)^{ij}\partial^\mu - i(\partial^\mu B_\mu)^{ij} g_{\alpha\beta}, && (3.10)\\
(N_2)^{ij}_{\alpha\beta} &= -(B_\mu B^\mu)^{ij} g_{\alpha\beta} - 2i(B_{\alpha\beta})^{ij}\,, && (3.11)
\end{aligned}$$

where we have used the notation $(X_\mu)^{ij} = -if^{ijk}X^k_\mu$. The operators T_1, T_2 and T_3 describe the θ-linear, that is the noncommutative vertices. They are more involved:

$$\begin{aligned}
(T_1)^{ij}_{\alpha\beta} = g'g^2\kappa_2\delta^{ij}\Big[& a(\overleftarrow{\partial_\mu}\theta^{\rho\sigma}f_{\rho\sigma}g_{\alpha\beta}\overrightarrow{\partial^\mu} - \overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\rho\sigma}\overrightarrow{\partial_\alpha}) && (3.12)\\
& -2(\overleftarrow{\partial_\beta}\theta_{\rho\alpha}f^{\mu\rho}\overrightarrow{\partial_\mu} - \overleftarrow{\partial^\nu}\theta^\rho{}_\alpha f_{\beta\rho}\overrightarrow{\partial_\nu} - \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}\overrightarrow{\partial^\mu} + \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\beta\rho}\overrightarrow{\partial_\alpha}\\
& +\overleftarrow{\partial_\mu}\theta_{\rho\beta}f^{\mu\rho}\overrightarrow{\partial_\alpha} - \overleftarrow{\partial^\nu}\theta^\rho{}_\beta f_{\alpha\rho}\overrightarrow{\partial_\nu} - \overleftarrow{\partial^\mu}\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}\overrightarrow{\partial_\sigma}\\
& +\overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\alpha\rho}\overrightarrow{\partial_\sigma}) + 2a(\overleftarrow{\partial_\rho}\theta^\rho{}_\alpha f_{\mu\beta}\overrightarrow{\partial^\mu} + \overleftarrow{\partial^\mu}\theta^\rho{}_\beta f_{\mu\alpha}\overrightarrow{\partial_\rho})\\
& -2(\overleftarrow{\partial_\mu}\theta_{\alpha\beta}f^{\mu\nu}\overrightarrow{\partial_\nu} - \overleftarrow{\partial^\mu}\theta_{\alpha\sigma}f_{\mu\beta}\overrightarrow{\partial^\sigma} - \overleftarrow{\partial^\sigma}\theta_{\beta\sigma}f_{\mu\alpha}\overrightarrow{\partial^\mu} + \overleftarrow{\partial_\rho}\theta^{\rho\sigma}f_{\alpha\beta}\overrightarrow{\partial_\sigma})\Big],\\
(T_2)^{ij}_{\alpha\beta} = g'g^2 i\kappa_2\Big[& a(-\overleftarrow{\partial_\mu}\theta^{\rho\sigma}g_{\alpha\beta}f_{\rho\sigma}(B^\mu)^{ij} - \theta^{\rho\sigma}f_{\rho\sigma}g_{\alpha\beta}(B^\mu)^{ji}\overrightarrow{\partial_\mu} && (3.13)\\
& +\overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\rho\sigma}(B_\alpha)^{ij} + \theta^{\rho\sigma}f_{\rho\sigma}(B_\beta)^{ji}\overrightarrow{\partial_\alpha} + \theta_{\rho\sigma}f^{\rho\sigma}(B_{\alpha\beta})^{ij})\\
& -2(-\overleftarrow{\partial_\beta}\theta_{\rho\alpha}f^{\mu\rho}(B_\mu)^{ij} - \theta_{\rho\beta}f^{\mu\rho}(B_\mu)^{ji}\overrightarrow{\partial_\alpha} + \overleftarrow{\partial_\nu}\theta_{\rho\alpha}f_\beta{}^\rho(B^\nu)^{ij}\\
& +\theta_{\rho\beta}f_\alpha{}^\rho(B^\nu)^{ji}\overrightarrow{\partial_\nu} + \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B^\mu)^{ij} + \theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B^\mu)^{ji}\overrightarrow{\partial_\sigma}\\
& -\overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\beta\rho}(B_\alpha)^{ij} - \theta^{\rho\sigma}f_{\alpha\rho}(B_\beta)^{ji}\overrightarrow{\partial_\sigma} - \overleftarrow{\partial_\mu}\theta_{\rho\beta}f^{\mu\rho}(B_\alpha)^{ij} - \theta_{\rho\alpha}f^{\mu\rho}(B_\beta)^{ji}\overrightarrow{\partial_\mu}\\
& +\overleftarrow{\partial_\mu}\theta^{\rho\sigma}g_{\alpha\beta}f^\mu{}_\rho(B_\sigma)^{ij} + \theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B_\sigma)^{ji}\overrightarrow{\partial^\mu} + \overleftarrow{\partial_\mu}\theta^\rho{}_\beta f_{\alpha\rho}(B^\mu)^{ij}\\
& +\theta_{\rho\alpha}f_\beta{}^\rho(B^\mu)^{ji}\overrightarrow{\partial_\mu} - \overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\alpha\rho}(B_\sigma)^{ij} - \theta^{\rho\sigma}f_{\beta\rho}(B_\sigma)^{ji}\overrightarrow{\partial_\alpha} + \theta^{\rho\sigma}f_{\alpha\rho}(B_{\beta\sigma})^{ij}\\
& +\theta_{\rho\beta}f^{\mu\rho}(B_{\mu\alpha})^{ij} + \theta^{\rho\sigma}f_{\beta\rho}(B_{\alpha\sigma})^{ji}\\
& +\theta_{\rho\alpha}f^{\mu\rho}(B^\mu{}_\beta)^{ji}) - 2a(\overleftarrow{\partial^\rho}\theta_{\rho\alpha}f_{\mu\beta}(B^\mu)^{ij} + \theta_{\rho\beta}f_{\mu\alpha}(B^\mu)^{ji}\overrightarrow{\partial^\rho}\\
& +\overleftarrow{\partial^\mu}\theta_{\rho\beta}f_{\mu\alpha}(B^\rho)^{ij} + \theta_{\rho\alpha}f_{\mu\beta}(B^\rho)^{ji}\overrightarrow{\partial^\mu} - \frac{1}{2}\theta_{\rho\sigma}f_{\alpha\beta}(B^{\rho\sigma})^{ij}\\
& -\frac{1}{2}\theta_{\alpha\beta}f_{\rho\sigma}(B^{\rho\sigma})^{ij}) - 2(-\overleftarrow{\partial^\mu}\theta_{\alpha\beta}f_{\mu\nu}(B^\nu)^{ij} - \theta_{\beta\alpha}f_{\mu\nu}(B^\nu)^{ji}\overrightarrow{\partial^\mu}\\
& +\overleftarrow{\partial^\mu}\theta_{\alpha\sigma}f_{\mu\beta}(B^\sigma)^{ij} + \theta_{\beta\sigma}f_{\mu\alpha}(B^\sigma)^{ji}\overrightarrow{\partial^\mu} + \overleftarrow{\partial^\rho}\theta_{\rho\beta}f_{\alpha\nu}(B^\nu)^{ij} + \theta_{\rho\alpha}f_{\beta\nu}(B^\nu)^{ji}\overrightarrow{\partial^\rho}\\
& -\overleftarrow{\partial_\rho}\theta_{\beta\sigma}f_{\alpha\beta}(B_\sigma)^{ij} - \theta^{\rho\sigma}f_{\beta\alpha}(B_\sigma)^{ji}\overrightarrow{\partial_\rho} + \theta_{\beta\sigma}f_{\alpha\nu}(B^{\nu\sigma})^{ij} + \theta_{\alpha\sigma}f_{\beta\nu}(B^\nu{}_\sigma)^{ji})\Big],\\
(T_3)^{ij}_{\alpha\beta} = g'g^2\kappa_2\Big[& a(\theta^{\rho\sigma}f_{\rho\sigma}(B_\mu B^\mu)^{ij}g_{\alpha\beta} - \theta^{\rho\sigma}f_{\rho\sigma}(B_\beta B_\alpha)^{ij}) && (3.14)\\
& -2(\theta_{\rho\alpha}f^{\mu\rho}(B_\beta B_\mu)^{ij} - \theta^\rho{}_\alpha f_{\beta\rho}(B_\nu B^\nu)^{ij} - \theta^{\rho\sigma}f_{\mu\rho}(B_\sigma B^\mu)^{ij}g_{\alpha\beta}\\
& +\theta^{\rho\sigma}f_{\beta\rho}(B_\sigma B_\alpha)^{ij} + (\alpha\leftrightarrow\beta\ \ i\leftrightarrow j))\\
& +2a(\theta_{\rho\alpha}f_{\mu\beta}(B^\rho B^\mu)^{ij} + 2\theta_{\rho\beta}f_{\mu\alpha}(B^\rho B^\mu)^{ji})\\
& -2(\theta_{\alpha\beta}f^{\mu\nu}(B_\mu B_\nu)^{ij} - \theta_{\alpha\sigma}f_{\mu\beta}(B^\mu B^\sigma)^{ij} - \theta_{\beta\sigma}f_{\mu\alpha}(B^\mu B^\sigma)^{ji} + \theta^{\rho\sigma}f_{\alpha\beta}(B_\rho B_\sigma)^{ij})\Big].
\end{aligned}$$

We do not write the matrix $W^{\alpha\beta,ab}$ explicitly as it is completely analogous to $V^{\alpha\beta,ij}$ up to the change $B^i_\mu \leftrightarrow G^a_\mu$.

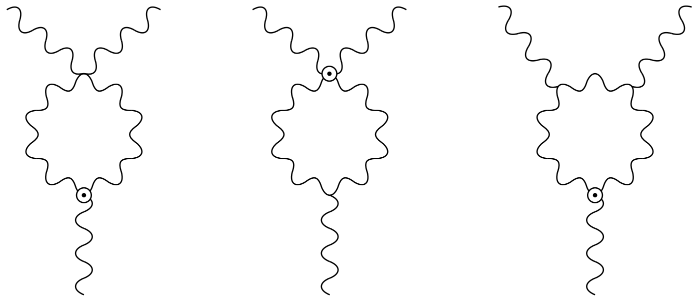

Figure 2: One-loop divergent corrections to the θ-3-vertex.

3.3 Divergences

We compute the divergences due to the $\mathrm{U}(1)_{\mathrm{Y}} - \mathrm{SU}(2)_{\mathrm{L}}$ part of the noncommutative action, S_2^θ. The result for $\mathrm{U}(1)_{\mathrm{Y}} - \mathrm{SU}(3)_{\mathrm{C}}$ is analogous and follows immediately. The one-loop effective action is

$$\begin{aligned}\Gamma^{(1)}_{\theta,2} &= \frac{i}{2}\mathrm{Tr}\,\log\left(\mathcal{I} + \Box^{-1}(N_1 + N_2 + T_1 + T_2 + T_3)\right) \qquad (3.15)\\ &= \frac{i}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\mathrm{Tr}\,\left(\Box^{-1}N_1 + \Box^{-1}N_2 + \Box^{-1}T_1 + \Box^{-1}T_2 + \Box^{-1}T_3\right)^n .\end{aligned}$$

For dimensional reasons, the divergences in θ-linear order are all of the form $\theta f B^2$. Consequently, from the sum (3.15) we need to extract and compute only terms that contain three external fields. A careful analysis gives that these terms are

$$\Gamma^{(1)}_{\theta,2} = \frac{i}{2}\mathrm{Tr}\,[(\Box^{-1}N_1)^2\Box^{-1}T_1 - \Box^{-1}N_1\Box^{-1}T_2 - \Box^{-1}N_2\Box^{-1}T_1]. \qquad (3.16)$$

As one can readily see, only the vertices obtain divergent contributions. For the θ-3-vertex, the diagrams which correspond to the traces in (3.15) are given in figure 2. Being written in terms of the field strengths, that is covariantly, (3.15) also contains the contributions to the θ-4-vertex and θ-5-vertex. We do not draw the corresponding diagrams: they can be easily obtained from figure 2 by adding external legs (in accordance with the Feynman rules). The divergent part of (3.16) is calculated in the momentum representation by dimensional regularization giving:

$$\begin{aligned}\mathrm{Tr}\,(\Box^{-1}N_1\Box^{-1}T_2) &= \frac{4i}{3(4\pi)^2\epsilon}g'g^2\kappa_2\\ &\times\Big[(6-2a)(\theta^{\rho\sigma}f_{\alpha\rho} + \theta_{\rho\alpha}f^{\sigma\rho})(B^{\alpha i}\partial_\mu\partial_\sigma B^{\mu i} - B^{\alpha i}\Box B^i_\sigma)\\ &+(3a-4)\theta^{\rho\sigma}f_{\rho\sigma}(B^{\nu i}\partial_\mu\partial_\nu B^{\mu i} - B^i_\mu\Box B^{\mu i})\Big], \qquad (3.17)\end{aligned}$$

$$\begin{aligned}\mathrm{Tr}\,(\Box^{-1}N_2\Box^{-1}T_1) &= \frac{4i}{3(4\pi)^2\epsilon}g'g^2\kappa_2\\ &\times\Big[(2a-6)(\theta^{\rho\sigma}f_{\alpha\rho} + \theta_{\rho\alpha}f^{\sigma\rho})(B^{\nu i}\partial_\sigma\partial^\alpha B^i_\nu + \partial_\sigma B^{\mu i}\partial^\alpha B^i_\mu)\\ &+\theta^{\rho\sigma}f_{\rho\sigma}(18-11a)(\partial_\nu B^{\nu i}\partial_\mu B^{\mu i} + B^i_\mu\Box B^{\mu i})\Big], \qquad (3.18)\end{aligned}$$

$$
\begin{aligned}
\mathrm{Tr}\,(\Box^{-1}N_1^2\Box^{-1}T_1) \;=\; & \frac{4i}{3(4\pi)^2\epsilon} g' g^2\kappa_2\Big[\theta^{\rho\sigma} f_{\rho\sigma}\Big((22-14a)B^i_\mu\Box B^{\mu i}\\
&+(15-10a)\partial_\nu B^{\mu i}\partial^\nu B^i_\mu\\
&+(3a-4)B^{\mu i}\partial_\mu\partial_\nu B^{\nu i}+(3-a)\partial_\mu B^{\nu i}\partial_\nu B^{\mu i}\Big)\\
&+(\theta^{\rho\sigma}f_{\alpha\rho}+\theta_{\rho\alpha}f^{\sigma\rho})\Big((2a-6)(B^i_\sigma\Box B^{\alpha i}-B^i_\sigma\partial^\alpha\partial_\mu B^{\mu i}\\
&+B^{\mu i}\partial_\sigma\partial^\alpha B^i_\mu-\partial_\sigma B^{\mu i}\partial_\mu B^{\alpha i})+(a-3)\partial_\mu B^{\alpha i}\partial^\mu B^i_\sigma\\
&+(3a-9)\partial_\sigma B^{\mu i}\partial^\alpha B^i_\mu\Big)\Big]. \qquad (3.19)
\end{aligned}
$$

Their sum, that is the complete divergent part due to the $\mathrm{U}(1)_\mathrm{Y}-\mathrm{SU}(2)_\mathrm{L}$ gauge boson interaction is

$$
\Gamma^{(1)}_{\mathrm{div}}=\frac{4}{3(4\pi)^2\epsilon}g'g^2\kappa_2(3-a)\theta^{\mu\nu}\int d^4x\,(\frac{1}{4}f_{\mu\nu}B^i_{\rho\sigma}B^{\rho\sigma i}-f_{\mu\rho}B^i_{\nu\sigma}B^{\rho\sigma i}). \qquad (3.20)
$$

Adding to this expression the divergences which come from the commutative part of the action, and also those induced by the $\mathrm{U}(1)_\mathrm{Y}-\mathrm{SU}(3)_\mathrm{C}$ mixing, we obtain the full result for the divergent one-loop effective action linear in θ:

$$
\begin{aligned}
\Gamma_{\mathrm{div}} \;=\; & \frac{11}{3(4\pi)^2\epsilon}\int d^4x B^i_{\mu\nu}B^{\mu\nu i}+\frac{11}{2(4\pi)^2\epsilon}\int d^4x G^a_{\mu\nu}G^{\mu\nu a}\\
&+\frac{4}{3(4\pi)^2\epsilon}g'g^2\kappa_2(3-a)\theta^{\mu\nu}\int d^4x(\frac{1}{4}f_{\mu\nu}B^i_{\rho\sigma}B^{\rho\sigma i}-f_{\mu\rho}B^i_{\nu\sigma}B^{\rho\sigma i})\\
&+\frac{6}{3(4\pi)^2\epsilon}g'g_S^2\kappa_3(3-a)\theta^{\mu\nu}\int d^4x(\frac{1}{4}f_{\mu\nu}G^a_{\rho\sigma}G^{\rho\sigma a}-f_{\mu\rho}G^a_{\nu\sigma}G^{\rho\sigma a}). \qquad (3.21)
\end{aligned}
$$

The divergent contribution due to $\mathrm{U}(1)_\mathrm{Y}$ solely vanishes, both the commutative and the noncommutative one.

3.4 Counter terms

It is clear from (3.21) that the divergences in the noncommutative sector vanish for the choice $a = 3$. Therefore one obtains that the noncommutative gauge sector interaction is not only renormalizable but finite. The renormalization is performed by adding counter terms to the Lagrangian. We obtain

$$
\begin{aligned}
\mathcal{L}+\mathcal{L}_{ct} \;=\; & -\frac{1}{4}f_{0\mu\nu}f_0{}^{\mu\nu}-\frac{1}{4}B_0{}^i_{\mu\nu}B_0{}^{\mu\nu i}-\frac{1}{4}G_0{}^a_{\mu\nu}G_0{}^{\mu\nu a}\\
&+g'^3\kappa_1\theta^{\mu\nu}\left(\frac{3}{4}f_{0\mu\nu}f_{0\rho\sigma}f_0{}^{\rho\sigma}-f_{0\mu\rho}f_{0\nu\sigma}f_0{}^{\rho\sigma}\right)\\
&+g_0'g_0^2\kappa_2\theta^{\mu\nu}\left(\frac{3}{4}f_{0\mu\nu}B_0{}^i_{\rho\sigma}B_0^{\rho\sigma i}-f_{0\mu\rho}B_0{}^i_{\nu\sigma}B_0^{\rho\sigma i}+c.p.\right)\\
&+g_0'(g_S)_0^2\kappa_3\theta^{\mu\nu}\left(\frac{3}{4}f_{0\mu\nu}G_0{}^a_{\rho\sigma}G_0^{\rho\sigma a}-f_{0\mu\rho}G_0{}^a_{\nu\sigma}G_0^{\rho\sigma a}+c.p.\right), \qquad (3.22)
\end{aligned}
$$

where the bare quantities are given as follows:

$$\mathcal{A}_0{}^{\mu} = \mathcal{A}^{\mu}\,, \qquad g'_0 = g'\,, \tag{3.23}$$

$$B_0{}^{\mu i} = B^{\mu i}\sqrt{1+\frac{44g^2}{3(4\pi)^2\epsilon}}\,, \qquad g_0 = \frac{g\,\mu^{\epsilon/2}}{\sqrt{1+\frac{44g^2}{3(4\pi)^2\epsilon}}}\,, \tag{3.24}$$

$$G_0{}^{\mu a} = G^{\mu a}\sqrt{1+\frac{22g_S^2}{(4\pi)^2\epsilon}}\,, \quad (g_S)_0 = \frac{g_S\,\mu^{\epsilon/2}}{\sqrt{1+\frac{22g_S^2}{(4\pi)^2\epsilon}}}\,. \tag{3.25}$$

In order to keep the constants κ_1, κ_2 and κ_3 in (3.22) unchanged under the renormalization procedure, i.e.

$$\kappa_1 = (\kappa_1)_0\,, \quad \kappa_2 = (\kappa_2)_0\,, \quad \kappa_3 = (\kappa_3)_0\,, \tag{3.26}$$

we obtain the following renormalization of the constants $C(\mathcal{R})$

$$\begin{aligned} C_1 &= (C_1)_0 + \frac{33}{18(4\pi)^2\epsilon}\,, \quad C_2 = (C_2)_0 - \frac{11}{18(4\pi)^2\epsilon}\,, \quad C_3 = (C_3)_0 - \frac{11}{18(4\pi)^2\epsilon}\,, \\ C_4 &= (C_4)_0 - \frac{143}{18(4\pi)^2\epsilon}\,, \quad C_5 = (C_5)_0 - \frac{121}{18(4\pi)^2\epsilon}\,, \quad C_6 = (C_6)_0 + \frac{110}{18(4\pi)^2\epsilon}\,. \end{aligned} \tag{3.27}$$

Finally, an important point is that the noncommutativity parameter θ need not be renormalized.

4. Discussion and conclusion

We have constructed a version of the standard model on the noncommutative Minkowski space which is one-loop renormalizable and finite in the gauge sector and in first order in the θ parameter. The renormalizability in the model was obtained by choosing six particle representations of the matter fields for the first generation of the SM as in table 1, and by fixing the arbitrariness in the θ-linear expansion terms in the Seiberg-Witten map.

The one-loop renormalizability of the NCSM gauge sector is certainly a very encouraging result from both theoretical and experimental perspectives. So far fermions have not been successfully included: the results on the renormalizability of noncommutative gauge theories with Dirac fermions are negative [13, 14] as a 4ψ-divergence always appears. This issue was analyzed in more details in papers [13, 15, 16] for noncommutative QED; it was obtained that renormalizability properties were the same for SW-expanded and SW-unexpanded theories. However, in the case of SU(N) or SU(3) $\otimes$ SU(2) $\otimes$ U(1) the unexpanded gauge theory cannot be consistently defined. Furthermore, our results show that the requirement of renormalizability fixes the SW freedom i.e. the corresponding parameter to $a = 1$ or $a = 3$ [27]. We hope that a similar procedure could be applicable to the fermionic sector of the theory.

Our result also has an important consequence on the phenomenological analysis of the $1 \to 2$ [9, 19, 23] and $2 \to 2$ [20 − 22] processes in elementary particle physics. Namely, in the gauge sector of the noncommutative standard model the above transitions contain triple gauge boson interactions induced by noncommutativity and, according to (3.21), they can be safely used further on. Since the triple gauge boson couplings have already been used in a number of phenomenological predictions to determine of the scale of noncommutativity [9, 10, 19, 21, 23], the regular behavior of these TGB interactions with respect to the one-loop renormalizability puts predictions from the NCSM gauge sector to a much firmer ground.

Experimentally, there are chances to detect, in the forthcoming experiments at LHC, the decays forbidden in the SM but kinematically allowed, like $Z \to \gamma\gamma$, and/or to find deviations of $\bar{f}f \to \gamma\gamma$, etc. scattering with respect to the standard model predictions. Finally, the discovery of forbidden decays, and/or measurements of differential cross section distributions deviating from the SM predictions, would certainly prove a violation of the SM as we know it at present and could serve as a possible indication/signal for space-time noncommutativity.

Acknowledgments

The authors want to thank J. Wess and J. Louis for fruitful discussions and the hospitality at DESY, Hamburg. The work of V.R. and M.B. is supported in part by the project 141036 of the Serbian Ministry of Science and by the DAAD grant A/06/05574. The work of J.T. is supported by the project 098-0982930-2900 of the Croatian Ministry of Science Education and Sports, and in part from A. von Humboldt Stiftung.

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